Lesson 4: GMM in Practice

Objectives

1. Estimate a Consumption Based Asset Pricing Model using GMM
2. Face the challenges of estimating a model in practice with real data

Theory

Consider the model with power utility and habit formation (Abel 1990) seen in class

$$M_{t+1} = \delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}$$

we do not need to log-linearize the model to estimate it using GMM. Let's use the pricing condition

$$\mathbb{E}_t[M_{t+1}R_{t+1}] = 1$$

Take expectations on both sides, and include instruments z_t to get more moments.

$$\begin{aligned}
\mathbb{E}_t[M_{t+1}R_{t+1} - 1] &= 0 \\
\mathbb{E}_t[M_{t+1}R_{t+1} - 1]z_t &= 0 \\
\mathbb{E}_t[(M_{t+1}R_{t+1} - 1)z_t] &= 0 \\
\mathbb{E}[\mathbb{E}_t[(M_{t+1}R_{t+1} - 1)z_t]] &= 0 \\
\mathbb{E}[(M_{t+1}R_{t+1} - 1)z_t] &= 0
\end{aligned}$$

GMM

$$g(\theta) = \mathbb{E}[(M_{t+1}(\theta)R_{t+1} - 1)z_t] = 0$$

E.g.

$$g([\gamma,\kappa,\delta]) = \mathbb{E}[\begin{pmatrix} \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1} - 1\right) \\ \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1} - 1\right)C_t \\ \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1} - 1\right)C_{t-1} \\ \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1} - 1\right)R_t \end{pmatrix}] = 0$$

GMM continued

$$g_T([\gamma,\kappa,\delta]) = \frac{1}{T}\sum_{t=1}^{T}\begin{pmatrix} \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1}-1\right) \\ \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1}-1\right)C_t \\ \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1}-1\right)C_{t-1} \\ \left(\delta\left(\frac{C_t}{C_{t-1}}\right)^{\kappa(\gamma-1)}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R_{t+1}-1\right)R_t \end{pmatrix} = 0$$

Data

Use quarterly data

- R_t is the return on the US market portfolio, obtain it here from Kenneth French's website https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/ftp/F-F_Research_Data_Factors_CSV.zip returns are at a monthly frequency, so you need to compute the quarterly returns. (Compound the monthly returns to get the quarterly return). Recall that R is already a gross return.
- C_t is the Real Personal Consumption Expenditures, obtain it here from FRED https://fred.stlouisfed.org/series/PCECC96.

This is Problem Set 3 due March 17 the day before the exam.