Production-Base Asset Pricing: a Primer

M2 104

Before we start

- For the exam, do not focus on the equations, focus on the intuition.
- Credit to Roberto Steri (U. of Luxembourg) for some of the material.

The Chicken and the Egg

No-arbitrage theory (e.g. Harrison and Kreps, 1979) implies asset pricing (AP) equations of the form

$$P_t = E_t[M_{t+1}X_{t+1}]$$

or, in terms of (gross) returns:

$$1 = E_t[M_{t+1}R_{t+1}]$$

where M_t is an SDF process, X_t is a payoff process, P_t is an asset price. Where do the joint statistical properties of X_t and M_{t+1} come from?

Consumption-based AP. Focus: **demand** of securities. "Standard" approach:

- endogenous M_t: investor's portfolio - consumption problem (e.g. $M_t = \beta \frac{u'(c_{t+1})}{u'(c_t)}$)
- exogenous X_t: stochastic process (e.g. Lucas' tree model)

Production-based AP. Focus: **supply** of securities. "Standard" approach:

- exogenous M_t: stochastic process
- endogenous X_t: optimal firm's policies (e.g. optimal investment)

The Chicken and the Egg

We can think of AP equations as determining today's consumption given asset prices and payoffs, rather than determining today's asset price in terms of consumption and payoffs.

Which is the chicken and which is the egg? Which variable is exogenous and which is endogenous?

The answer is, neither, and for many purposes, it doesn't matter.

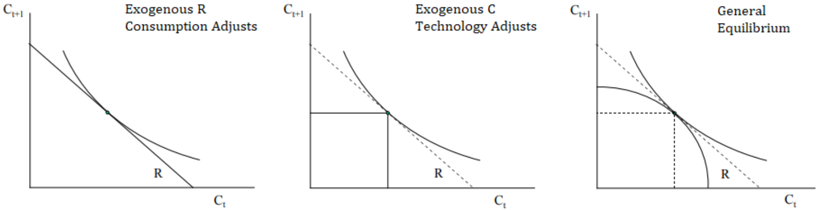
- e.g. cross-sectional studies: M_t can be thought as a function of aggregate variables (market return, aggregate consumption), so it is plausible to hold the properties of the discount factor constant as we study one individual asset after another
- e.g. portfolio-consumption studies: typically restrict the number of assets, e.g. just an interest rate, and study the time-series evolution of aggregate or individual consumption

The Chicken and the Egg

Both approaches are useful and can co-exist: they are looking at two sides of the same coin

Next step: complete solution of the model economy ("actual" general equilibrium)

E.g. Market clearing requires asset prices to adjust to equate marginal rate of substitution (MRS) and marginal rate of transformation (MRT)



E.g. Consumption and Rates of Returns with Different Technologies

The Chicken and the Egg

Challenges:

- specifying a sensible consumption side of the economy (e.g. equity premium puzzle)
- which corporate policies matter? Do production/investment suffice?

To start with, there is nothing wrong in adopting one of the following strategies:

- Form a statistical model of bond and stock returns, solve the optimal consumption portfolio decision. Use the equilibrium consumption values in $P_t = E_t[M_{t+1}X_{t+1}]$
- Form a statistical model of the consumption process (or SDF), calculate asset prices and returns directly from the basic pricing equation $P_t = E_t[M_{t+1}X_{t+1}]$
- Form a completely correct general equilibrium model, including the production technology, utility function and specification of the market structure. Derive the equilibrium consumption and asset price process, including $P_t = E_t[M_{t+1}X_{t+1}]$ as one of the equilibrium conditions

Motivation/Questions

Historically, the consumption-based approach came first. Why is the production-based approach emerging?

- after all, cash flows do not grow on trees
- cash flows are supplied by producers based on investment and other corporate policies
- cash flows are property of security's owners (e.g. equities, bonds)

Measurement? Maybe better able to measure production-side variables

Stock prices can forecast future economic activity (Y,I,...)

Today' Outline

Objective: link asset prices to investment dynamics

List of topics:

- Neoclassical Investment
- Stock Returns and Investment Returns
- Production-Based M
- Production-Based β's
- Production-Based Asset-Pricing in General Equilibrium (overview)
- Corporate Policies and Asset Prices (overview)

Neoclassical Firm

Assume no arbitrage: $\mathrm{E}_t[M_{t+1}(1+R_{t+1})] = 1$

Add more structure to stock returns by linking to firms' investment decisions

Use marginal rates of transformation to pin down asset returns

Take M (e.g., marginal rates of substitution) as exogenous

Need to model the firm's intertemporal investment-payout problem...

Neoclassical Investment

Assume perfect competition and no manager - shareholder conflicts

Assume that firm maximizes the present discounted value of dividends to shareholders:

$$V_t = \max_{\mathbf{X}_t} \mathsf{E}_t \left[\sum_{j=0}^{\infty} M_{t,t+j} D_{t+j} \right]$$

$\mathbf{X}_t$ can include optimal investment, output, labor, and financing choices

subject to constraints

- sources and uses of funds
- production technology $F(K, L, ...)$
- capital accumulation

Implication of firm's problem for asset prices?

Neoclassical Investment

Stock return:

$$1 + R_{t+1} \equiv \frac{V_{t+1}}{V_t - D_t} = \frac{P_{t+1} + D_{t+1}}{P_t}$$

where V is the *cum*-dividend value and P is the *ex*-dividend value

Since V and D are determined by the optimal firm choices, returns do not grow on trees, i.e. $R_{t+1} = R(I, K, Y, L, B, ...)$

Connect asset prices to other macroeconomic fundamentals other than consumption

Neoclassical Investment

Modigliani-Miller assumptions hold so that we can ignore capital structure decisions (more on this later)

Neoclassical framework example with adjustment costs

Operating profits (substituting out the foc wrt to L_t):

$$\Pi(K_t, A_t) = \max_{L_t} \{P_t A_t * F(K_t, L_t) - W_t L_t - \xi\}$$

- Normalize the price of goods P_t to 1
- A_t is an exogenous productivity shock
- W_t is the wage rate and L_t is labor hours
- ξ is the fixed cost of production
- $F(\cdot, \cdot)$ is the production technology

Neoclassical Investment

Firm's problem:

$$V_0 = \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} M_{0,t} D_t \right]$$

subject to

$$\begin{aligned} D_t &= \Pi(K_t, A_t) - I_t - \Phi(I_t, K_t) \\ K_{t+1} &= (1-\delta) K_t + I_t \end{aligned}$$

where $\phi(\cdot, \cdot)$ capture convex capital adjustment costs

We can rewrite the problem recursively as

$$V_t = \max_{I_t, K_{t+1}} \{ D_t + \mathrm{E}_t [M_{t,t+1} V_{t+1}] \}$$

subject to the constraints above

Neoclassical Investment

We can write the problem more succinctly as:

$$V_t = \max_{I_t, K_{t+1}} \left\{ \begin{array}{c} \Pi(K_t, A_t) - I_t - \Phi(I_t, K_t) + \mathrm{E}_t[M_{t,t+1} V_{t+1}] \\ + q_t(I_t + (1-\delta)K_t - K_{t+1}) \end{array} \right\}$$

where q is the Lagrange multiplier and $V_t = V(K_t, A_t)$

Combining first-order and envelope conditions:

$$\begin{aligned} q_t &= 1 + \Phi_I(I_t, K_t) \\ 1 &= \mathrm{E}_t \left[M_{t,t+1} \frac{\Pi_K(K_{t+1}, A_{t+1}) - \Phi_K(I_{t+1}, K_{t+1}) + q_{t+1}(1-\delta)}{q_t} \right] \end{aligned}$$

Invest up to the point where the marginal cost equals the expected marginal benefit

Neoclassical Investment

Suppose that the production technology and adjustment costs are, respectively, linear and homogenous of degree one in both arguments (a.k.a. Hayashi conditions):

$$\begin{aligned}\Pi(K,A) &= \Pi_K(K,A) * K \\ \Phi(I,K) &= \Phi_K(I,K) * K + \Phi_I(I,K) * I\end{aligned}$$

Define $Q \equiv P_t/K_t$ (average or Tobin's Q)

With the linear homogeneity assumption, Hayashi shows that $q = Q$ (equivalence of shadow value and market value)

Neoclassical Investment

Proof: Rewrite the value of firm:

$$
\begin{aligned}
V_0 &= \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} M_{0,t} \left(\begin{array}{c} \Pi(K_t, A_t) - I_t - \Phi_K(I_t, K_t) K_t - \\ \Phi_I(I_t, K_t) I_t + q_t I_t + q_t (1-\delta) K_t - q_t K_{t+1} \end{array} \right) \right] \\
&= \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} M_{0,t} \left(\begin{array}{c} (\Pi_K(K_t, A_t) - \Phi_K(I_t, K_t) + q_t (1-\delta) K_t \\ +(q_t - 1 - \Phi_I(I_t, K_t)) I_t - q_t K_{t+1} \end{array} \right) \right]
\end{aligned}
$$

Using the foc $q_t - 1 - \Phi_I(I_t, K_t) = 0$:

$$
\begin{aligned}
V_0 &= \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\sum_{t=0}^{\infty} M_{0,t} \left(\begin{array}{c} (\Pi_K(K_t, A_t) - \Phi_K(I_t, K_t) + q_t (1-\delta)) K_t \\ -q_t K_{t+1} \end{array} \right) \right] \\
&= \max_{\{I_t, K_{t+1}\}_{t=0}^{\infty}} \mathrm{E}_0 \left[\begin{array}{c} \cdots - M_{0,t} q_t K_{t+1} + \\ \mathrm{E}_t [M_{0,t+1} (\Pi_K(K_{t+1}, A_{t+1}) - \Phi_K(I_{t+1}, K_{t+1}) + \\ q_{t+1}(1-\delta)) K_{t+1}] - M_{0,t+1} q_{t+1} K_{t+2} + \cdots \end{array} \right]
\end{aligned}
$$

Neoclassical Investment

Rearranging the foc wrt to capital and multiplying both sides by $M_{0,t} * K_{t+1}$:

$$0 = -M_{0,t} q_t K_{t+1} + \mathrm{E}_t \left[\begin{array}{c} M_{0,t+1}(\Pi_K(K_{t+1}, A_{t+1}) - \Phi_K(I_{t+1}, K_{t+1}) \\ +q_{t+1}(1-\delta))K_{t+1} \end{array} \right]$$

Plugging this in the value of firm:

$$V_0 = \Pi(K_0, A_0) - \Phi_K(I_0, K_0)K_0 + q_0(1-\delta)K_0 - \lim_{t\to\infty} E_0[M_{0,t+1} q_{t+1} K_{t+2}]$$

Ruling out rational bubbles (transversality condition):

$$\begin{aligned} V_0 &= \Pi(K_0, A_0) - [\Phi(I_0, K_0) - \Phi_I(I_0, K_0) I_0] + q_0(1-\delta)K_0 \\ &= \Pi(K_0, A_0) - \Phi(I_0, K_0) + (q_0 - 1)I_0 + q_0(1-\delta)K_0 \\ &= \Pi(K_0, A_0) - \Phi(I_0, K_0) - I_0 + q_0[I_0 + (1-\delta)K_0] \\ &= D_0 + q_0 K_1 \end{aligned}$$

thus, $P_0 \equiv V_0 - D_0 = q_0 K_1$ ■

Neoclassical Investment

Implications of the Hayashi conditions are important for the empirical investment literature

The foc's in the linear homogenous case can be expressed as:

$$\begin{aligned} q_t &= \frac{P_t}{K_{t+1}} \quad (\equiv \text{Tobin's Q}) \\ q_t &= 1 + \Phi_I(I_t, K_t) \end{aligned}$$

First equation says that marginal and average values of capital are equal - the latter is observable

The second equation implies that Q is also a sufficient statistic for investment

With no adjustment costs, $P = K$ (market value = book value)

How well does the Q-theory hold in the data?

Stock Returns and Investment Returns

Define the investment return as:

$$1 + R^I_{t+1} \equiv \frac{\Pi_K(K_{t+1}, A_{t+1}) - \Phi_K(I_{t+1}, K_{t+1}) + q_{t+1}(1-\delta)}{q_t}$$

The stock return is:

$$1 + R^D_{t+1} \equiv \frac{P_{t+1} + D_{t+1}}{P_t}$$

Hayashi conditions are sufficient for $R^I = R^D$ state-by-state

proof: follows almost directly from the equivalence of q and Q

Stock Returns and Investment Returns

Multiply the numerator and denominator of $1 + R^I$ by K_{t+1}:

$$\frac{\Pi_K(K_{t+1}, A_{t+1})K_{t+1} - \Phi_K(I_{t+1}, K_{t+1})K_{t+1} + q_{t+1}(1-\delta)K_{t+1}}{q_t K_{t+1}}$$

Thus, in the denominator we have P_t

Rewrite the numerator:

$$\begin{aligned}
&= \Pi(K_{t+1}, A_{t+1}) - (\Phi(I_{t+1}, K_{t+1}) - \Phi_I(I_{t+1}, K_{t+1})I_{t+1}) + q_{t+1}(1-\delta)K_{t+1} \\
&= \Pi(K_{t+1}, A_{t+1}) - \Phi(I_{t+1}, K_{t+1}) + (q_{t+1} - 1)I_{t+1} + q_{t+1}(1-\delta)K_{t+1} \\
&= \Pi(K_{t+1}, A_{t+1}) - \Phi(I_{t+1}, K_{t+1}) - I_{t+1} + q_{t+1}(I_{t+1} + (1-\delta)K_{t+1}) \\
&= D_{t+1} + q_{t+1}K_{t+2} \\
&= D_{t+1} + P_{t+1}
\end{aligned}$$

Thus,

$$1 + R^I_{t+1} = \frac{P_{t+1} + D_{t+1}}{P_t} \equiv 1 + R^D_{t+1}$$

Stock Returns and Investment Returns

Does the equivalence of stock and investment returns hold in the data?

Need to impose more structural assumptions to take to data

Assume quadratic adjustment costs:

$$\Phi(I_t, K_t) = \frac{\kappa}{2} * \left(\frac{I_t}{K_t} - \lambda\right)^2 I_t$$

Return on investment is a function of the current and lagged I/K (and the model parameters)

Cochrane (1991) construction of aggregate investment return:

- Input the aggregate investment rate series from data into $R^I(I_t/K_t, I_{t-1}/K_{t-1})$
- Set $\delta = .10$
- Calibrate α and Π_K to match mean and vol of aggregate stock returns

Aggregate stock returns: value-weighted NYSE portfolio

Stock Returns and Investment Returns

Let's compare the series of investment and stock returns

Data are quarterly, 1947:1–1987:4. Annual returns are overlapping quarterly observations. All returns are expressed as percentages. Autocorrelations are calculated from single regression slope coefficients. Stock returns are the CRSP value weighted portfolio deflated by the CPI; the investment return is constructed from gross fixed investment data.

			Quarterly		Annual	
		Investment/ Capital Ratio	Investment Return	Stock Return	Investment Return	Stock Return
Mean		0.137	1.70	1.70	7.33	7.33
Standard deviation		0.009	3.42	7.24	9.37	15.53
Autocorrelations	1	0.90	0.45	0.11		
(by lag, in quarters)	2	0.71	0.10	0.04		
	3	0.49	−0.06	−0.04		
	4	0.28	−0.19	−0.03	−0.18	−0.07
	5	0.12	−0.23	−0.10		
	6	0.00	−0.13	−0.07		
	8	−0.17	−0.18	0.06	−0.20	−0.07
	12	−0.36	−0.19	0.04	−0.23	0.07

source: Cochrane (1991)

Investment return dynamics are similar to stock return dynamics

Stock Returns and Investment Returns

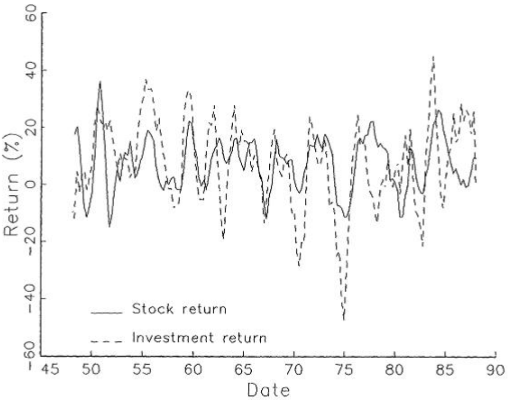


Figure 2. Quarterly observations of annual (from t − 4 to t) real returns on the value weighted NYSE portfolio, and annual investment returns.

source: Cochrane (1991)

Consistent with theory, investment returns co-move with stock returns

Stock Returns and Investment Returns

Panel C. Annual Returns with No Overlap (First Quarter to First Quarter, etc.)
Stock Return ($t-4 \rightarrow t$) = $\alpha + \beta$ Investment Return ($t-4 \rightarrow t$) + $\varepsilon(t)$

Data sample	t-stat.	% p value[a]	Correlation of of stock, inv. return	Std. error of correlation
First quarter	2.885	0.634	0.449	0.128
Second quarter	2.578	1.384	0.407	0.139
Third quarter	1.851	7.173	0.306	0.141
Fourth quarter	2.569	1.412	0.404	0.137

source: Cochrane (1991)

Investment returns co-move with stock returns in a economically significant manner

Stock Returns and Investment Returns

Hayashi conditions are stringent ones

However, linking stock returns to investment is not limited to the linear homogenous case

Hayashi setting provides a simple analytical benchmark for thinking about investment-based stock returns

Production-based M

So far, M is exogenous, i.e. taken as given by the firm from the investor maximization problem

Can we use the production possibilities frontier to infer M?

- Link M to marginal rates of transformation (as opposed to MRS)

"Standard" models do not allow firms to adjust output across states

$$Y_{t+1}(s) = \Theta_{t+1}(s) F(K_{t+1})$$

where $\Theta_{t+1}(s)$ is exogenous and K_{t+1} is predetermined (time-to-build)

Only allows for intertemporal adjustment $\Rightarrow$ cannot back out state prices

Modification: allow for state-contingent production plans (Cochrane, Belo)

$$Y_{t+1}(s) = \epsilon_{t+1}(s) F(K_{t+1})$$

where firms can choose $\epsilon_{t+1}(s)$ subject to

$$\mathrm{E}_t \left[\left(\frac{\epsilon_{t+1}}{\Theta_{t+1}} \right)^{\alpha} \right]^{\frac{1}{\alpha}} \leq 1$$

Production-based M

$\alpha > 1$ dictates the ability the firm has in adjusting output across states

Creates smooth production possibilities frontier - well-defined MRT

Limiting case: $\alpha \to \infty \Rightarrow \epsilon_t = \Theta_t$ (no adjustment across states)

Firm's problem:

$$V(\mathbf{X}_{it}) = \max_{I_{it}, \epsilon_{it+1}} \{D_{it} + \mathrm{E}_t[M_{t+1} V(\mathbf{X}_{it+1})]\}$$

subject to

$$\begin{aligned} D_{it} &= P_{it} Y_{it} - I_{it} \\ Y_{it} &= \epsilon_{it} F^i(K_{it}) \\ 1 &\geq \mathrm{E}_t \left[\left(\frac{\epsilon_{it+1}}{\Theta_{it+1}} \right)^{\alpha} \right]^{\frac{1}{\alpha}} \\ K_{it+1} &= (1 - \delta_i) K_{it} + I_{it} \end{aligned}$$

where $\mathbf{X}_{it} \equiv [K_{it}, \epsilon_{it}, P_{it}, Z_{it}]$

Production-based M

The foc with respect to ϵ_{it+1} is

$$\frac{\epsilon_{it+1}}{\epsilon_{it}} = \varphi_{it}^{\frac{1}{1-\alpha}} \left(\frac{M_{t+1}P_{it+1}}{P_{it}}\right)^{\frac{1}{\alpha-1}} \left(\frac{\Theta_{it+1}}{\Theta_{it}}\right)^{\frac{\alpha}{\alpha-1}}$$

where $\varphi_{it} = \mathrm{E}_t[M_{t+1}P_{it+1}/P_{it}]/\mathrm{E}_t[(\epsilon_{it+1}/\epsilon_{it})^{\alpha-1}(\Theta_{it+1}/\Theta_{it})^{-\alpha}]$

Demand-side: firm chooses higher productivity in states where output is more valuable (high M, P_{it})

Supply-side: firm chooses higher productivity in states where technology is more efficient (high Θ_{it})

Production-based M

Rearrange the foc to obtain M:

$$M_{t+1} = \varphi_{it} \left(\frac{P_{it+1}}{P_{it}}\right)^{-1} \left(\frac{\epsilon_{it+1}}{\epsilon_{it}}\right)^{\alpha-1} \left(\frac{\Theta_{it+1}}{\Theta_{it}}\right)^{-\alpha}$$

Using the fact that $F(K_{it+1})$ is predetermined, rewrite in terms of Y:

$$M_{t+1} = \overline{\varphi}_{it} \left(\frac{P_{it+1}}{P_{it}}\right)^{-1} \left(\frac{Y_{it+1}}{Y_{it}}\right)^{\alpha-1} \left(\frac{\Theta_{it+1}}{\Theta_{it}}\right)^{-\alpha}$$

where $\overline{\varphi}_{it} = \mathrm{E}_t[M_{t+1}P_{it+1}/P_{it}]/\mathrm{E}_t[(Y_{it+1}/Y_{it})^{\alpha-1}(\Theta_{it+1}/\Theta_{it})^{-\alpha}]$

But, Θ_i is not observable...how to identify this?

Belo (2010) assumes a single factor structure:

$$\alpha\Delta\log(\Theta_{it}) = \lambda_i F_t$$

where the normalization $\lambda_1 = 1$ is used (good 1 is the numeraire)

Production-based M

Take the Euler equation of any two arbitrary producers 1 and 2

- Common productivity factor

$$F_t = (\lambda - 1)^{-1}[\gamma_{2t-1} - \gamma_{1t-1}] + b^p(\Delta p_{2t} - \Delta p_{1t}) + b^y(\Delta y_{2t} - \Delta y_{1t})$$

- Stochastic discount factor

$$M_t = \kappa_{t-1} \exp[-b^p(\Delta p_{2t} - \Delta p_{1t}) - b^y(\Delta y_{2t} - \Delta y_{1t}) + (\alpha - 1)\Delta y_{1t}]$$

where $b^p \equiv 1/(1-\lambda)$, $b^y \equiv (\alpha-1)/(\lambda-1)$, $\lambda \equiv \lambda_2$, $\gamma_{it} \equiv \log(\overline{\varphi_{it}})$, $\kappa_t \equiv \exp[(\lambda-1)^{-1}(\lambda\gamma_{1t} - \gamma_{2t})]$

More generally, if Θ_i loads on K common factors, then we need $K+1$ sectors to identify Θ_i

κ_{t-1} is predetermined and therefore doesn't matter for excess returns (covariance risk)

We have a production-based M (no consumption)

Production-based M

Empirical test: what to use for sectors 1 and 2?

Belo uses durable production for sector 1 and nondurable production for sector 2

Estimate α, λ using GMM

$$0 = \mathrm{E}\left[M_{t+1} R^e_{it+1} \mathbf{Z}_t\right]$$

where $\mathbf{Z}_t$ is a Ix1 vector of instruments

Use various test portfolios for R's

Production-based M

Table 2
GMM estimation of the production-based model.

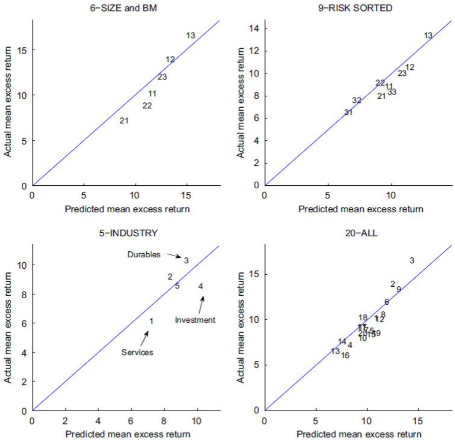
	6-Size-BM		9-Risk		5-Ind		20-All	
	1st	2nd	1st	2nd	1st	2nd	1st	2nd
Parameters								
α	1.04	1.02	1.05	1.00	1.16	1.12	1.05	1.04
	[0.25]	[0.08]	[0.20]	[0.08]	[0.25]	[0.17]	[0.15]	[0.05]
λ	0.96	0.97	0.96	0.97	0.94	0.95	0.96	0.97
	[0.04]	[0.01]	[0.03]	[0.01]	[0.05]	[0.03]	[0.02]	[0.01]
Diagnostics								
R^2	95.12		84.08		21.81		83.29	
MAE	1.15		0.66		0.94		0.92	
J-test	12.70	12.64	11.89	12.93	12.20	12.16	23.61	23.62
p-Value (J)	24.07	24.47	75.13	67.79	14.25	14.43	96.73	96.72

source: Belo (2010)

Model fits data fairly well - model is not rejected by the J-test and R^2's are sizable

Given these parameter estimates, high M states are when price (output) of nondurables is low relative to durables

Production-based M

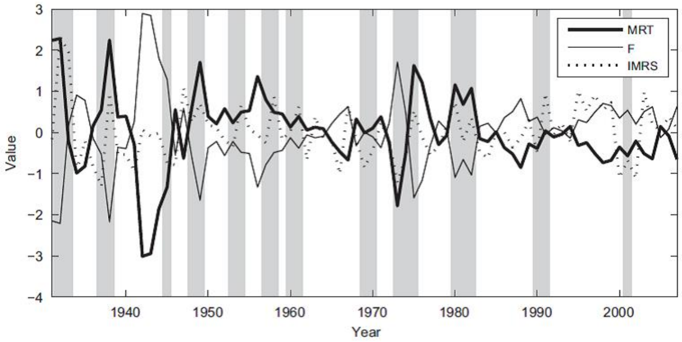


source: Belo (2010)

Pricing error for $i = \mathrm{E}\left(R_i^e\right)^{observed} - \mathrm{E}\left(R_i^e\right)^{predicted}$

Most test assets lie on the 45 degree line

Production-based M



source: Belo (2010)

Recessions occur when productivity (F) is low

Comparing the production-based M with the consumption-based M from Yogo (2006)

Production-based M

Production-based approach allows us to infer M from firm's foc's

Need to modify standard tech to allow for state-contingent plans

In a perfectly competitive economy, the consumption- and production-based M's are equal state-by-state

Therefore, these are complimentary approaches

Production-based βs

Can we link risk exposures (β's) to firm production choices?

Why do certain firms earn higher expected returns than others?

Firms with high B/M earn significantly higher returns than low B/M (value premium ≈ 5%)

With law of one price, we have the beta representation:

$$\begin{aligned} E[1+R_i] &= \kappa + \left(\frac{\text{Cov}(M, R_i)}{\text{Var}(M)}\right)\left(\frac{\text{Var}(M)}{E(M)}\right) \\ &= \kappa + \beta_i \lambda \end{aligned}$$

Risk-based explanation: high B/M firms have higher β's (quantity of risk)

Intuition: high B/M firms do poorly during "bad times"

Is this intuition consistent with the neoclassical investment framework? data?

Production-based βs

Zhang (2005) builds a neoclassical investment framework to explain the value premium

Assume an exogenous reduced-form M:

$$\begin{aligned} \log(M_{t+1}) &= \log(\beta) - \gamma_t(x_{t+1} - x_t) \\ \gamma_t &= \gamma_0 + \gamma_1(x_t - \overline{x}) \end{aligned}$$

where x_t is aggregate productivity

Continuum of competitive firms that produce a homogeneous good

Profit function for firm j:

$$\Pi_{jt} = e^{x_t + z_{jt} + p_t} K_{j,t}^{\alpha} - F$$

where log aggregate and idiosyncratic productivity follow:

$$\begin{aligned} x_{t+1} &= \overline{x}(1 - \rho_x) + \rho_x x_t + \sigma_x \epsilon_{t+1}^x \\ z_{jt+1} &= \rho_z z_{jt} + \sigma_z \epsilon_{jt+1}^z \end{aligned}$$

Production-based βs

Industry demand:

$$P_t = Y_t^{\eta}$$

where η is the inverse price elasticity demand

Firm's problem (drop subscript i for simplicity):

$$V_t = \max_{I_t} \{D_t + \mathrm{E}_t[M_{t+1} V_{t+1}]\}$$

subject to

$$\begin{aligned} D_t &= \Pi_t - I_t - H(I_t, K_t) \\ K_{t+1} &= (1-\delta)K_t + I_t \end{aligned}$$

and $V_t \equiv V(K_t, Z_t; X_t, P_t)$

Production-based βs

Capital adjustment costs are quadratic and asymmetric:

$$H(I_t, K_t) = \frac{\theta_t}{2}\left(\frac{I_t}{K_t}\right)^2 K_t$$

where $\theta^- > \theta^+$ and $\theta_t = \theta^+ \chi_{\{I_t \geq 0\}} + \chi_{\{I_t < 0\}}$

Asymmetry captures investment irreversibility: more costly to disinvest than invest

Let μ_t represent the cross-sectional distribution of firms

Market clearing:

$$Y_t = \int\int e^{x_t + z_{jt}} K_{j,t}^{\alpha} \mu_t(dk, dz; x)$$

approximate distribution using a bounded rationality approach (Krusell-Smith's algorithm)

Production-based βs

Assume the existence of a riskfree rate, then expected returns:

$$\mathrm{E}_t[1 + R_{jt+1}] = R_{ft} + \beta_{jt}\lambda_t$$

The stock return is defined as:

$$R_{jt+1} = \frac{V_{jt+1}}{V_{jt} - D_t}$$

In the model, value (growth) firms: low (high) realizations of z

Value firms (high K/V) in the the model have higher βs - why?

Production-based βs

Modified standard neoclassical framework to incorporate:

- Asymmetric costs $\longrightarrow$ countercyclical value premium
- Time-varying risks $\longrightarrow$ propagates risk dynamics

Intuition for the countercyclical value premium

- In bad times:
  value firms - burdened with more unproductive $K \Rightarrow \downarrow I$ more $\Rightarrow$ higher adj costs $\Rightarrow$ higher risk
- In good times:
  growth firms - more productive $K \Rightarrow \Rightarrow \uparrow I$ more $\Rightarrow$ higher adj costs $\Rightarrow$ higher risk

Intuition for positive value premium:

- Asymmetric adj costs - $\downarrow I$ is more costly than $\uparrow I$
- Countercyclical risk aversion - value firms do worse in high M states

Production-based βs

Group I					Group II			Group III				
α	δ	ρ_x	σ_x	η	β	γ_0	γ_1	θ^-/θ^+	θ^+	ρ_z	σ_z	f
0.30	0.01	$0.95^{1/3}$	0.007/3	0.50	0.994	50	−1000	10	15	0.97	0.10	0.0365

source: Zhang (2005)

Monthly calibration

z is calibrated to match the dispersion in K/V

Production-based β's

Moments	Model	Data
Average annual Sharpe ratio	0.41	0.43
Average annual real interest rate	0.022	0.018
Annual volatility of real interest rate	0.029	0.030
Average annual value-weighted industry return	0.13	0.12–0.14
Annual volatility of value-weighted industry return	0.27	0.23–0.28
Average volatility of individual stock return	0.286	0.25–0.32
Average industry book-to-market ratio	0.54	0.67
Volatility of industry book-to-market ratio	0.24	0.23
Annual average rate of investment	0.135	0.15
Annual average rate of disinvestment	0.014	0.02

source: Zhang (2005)

Model is calibrated to be consistent with wide range of moments

Production-based βs

	Panel A: Data and Benchmark						Panel B: Comparative Statics					
	Data			Benchmark			Model 1			Model 2		
	m	β	σ	m	β	σ	m	β	σ	m	β	σ
HML	**4.68**	0.14	0.12	**4.87**	0.43	0.12	**2.19**	0.09	0.04	**2.54**	0.11	0.04
Low	0.11	1.01	0.20	0.09	0.85	0.23	0.08	0.95	0.30	0.08	0.94	0.30
2	0.12	0.98	0.19	0.10	0.92	0.24	0.09	0.97	0.31	0.09	0.97	0.31
3	0.12	0.95	0.19	0.10	0.95	0.25	0.09	0.99	0.31	0.09	0.98	0.31
4	0.11	1.06	0.21	0.11	0.98	0.26	0.09	1.00	0.32	0.10	0.99	0.31
5	0.13	0.98	0.20	0.11	1.01	0.27	0.10	1.00	0.32	0.10	1.00	0.32
6	0.13	1.07	0.22	0.12	1.04	0.28	0.10	1.01	0.32	0.10	1.01	0.32
7	0.14	1.13	0.24	0.12	1.08	0.28	0.10	1.02	0.32	0.10	1.02	0.32
8	0.15	1.14	0.24	0.12	1.12	0.30	0.10	1.03	0.33	0.11	1.04	0.33
9	0.17	1.31	0.29	0.13	1.18	0.31	0.11	1.04	0.33	0.11	1.05	0.33
High	0.17	1.42	0.33	0.15	1.36	0.36	0.11	1.07	0.34	0.12	1.08	0.34

source: Zhang (2005)

Model 1 ~ symmetric adj costs, constant price of risk; model 2 ~ asymmetric adj costs, constant price of risk

Time-varying risks are key for the value premium

Motivation and Questions

Merge the consumption- and production-based approaches in a general equilibrium setting

Questions:

- Investors can smooth consumption through savings technology - asset pricing implications still hold?
- Jointly reconcile business cycle fluctuations and asset return data?
- Role of macroeconomic distortions (taxes, wage rigidities, etc) on asset prices?
- How does using asset pricing data impact welfare cost calculations?

Mainstream Approach

- Euler equation (consumption/savings):

$$1 = E_t \left[M_{t+1} R_{t+1} \right]$$

- Assume complete markets: $M_{t+1} = F(C_t, C_{t+1}, \cdot)$
- Link asset prices to the production-side in general equilibrium:
  a. $C_t = C(S_t) \Rightarrow M_{t+1} = G(S_t, S_{t+1}, \cdot)$ where $S_t = [K_t, Z_t, ...]$
  b. $R_{t+1} = (D_{t+1} + P_{t+1})/P_t \Leftrightarrow R^I_{t+1} = MPK_{t+1} - \delta + \Theta_{t+1}$
  c. $P =$ marginal $q =$ average Q

* need Hayashi's conditions for [b] & [c] to hold

Mainstream Approach

* Take a particular consumption-based model
  1. Preferences (Habits, Epstein-Zin, etc)
  2. exogenous dividend & consumption processes
* build a production-based model with the same preferences
* if we find policies $C(S_t)$ and $D(S_t)$ that replicate [2.], then asset pricing implications will be identical

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Beyond Investment: Corporate Policies and Asset Prices

Many predictors of returns are firm-level variables (e.g. cash, hiring rates, etc.)

The production-based literature is considering models with other corporate policies besides real investment

In particular, although the corporate finance literature and the asset pricing literature are still largely separate, these are the two sides of the same coin

- e.g. financial constraints, dynamic contracting
- e.g. industrial organization (IO), imperfect competition, markups
- e.g. feedback effects of stock market prices on corporate policies

Promising avenue for future research! For references, see the programs at

`https://sites.google.com/site/coapconference/home`

Conclusion

Link asset prices to production in neoclassical investment framework

Hayashi conditions imply equivalence between investment and stock returns

Infer M from firm foc's when production is allowed to be state-contingent

Neoclassical framework with time-varying risks and asymmetric adjustment costs can explain the value premium

Room for working at the interface of corporate policies and asset prices

What happens when we merge the consumption- and production-based approaches?

The Classic Corner

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