Problem Set 1

Empirical Asset Pricing

M2 104

Paris Dauphine - PSL

In this problem set you will study the properties of the bias correction presented in ***Stambaugh (1999) Predictive Regressions, Journal of Financial Economics***. The problem set together with the code needs to be emailed to juan.imbet@dauphine.psl.eu *before* February 4 2024 23:59. You can solve the problem sets in groups of maximum *3* people.

Setup

Consider the following system of predictive regressions

$$y_{t+1} = \alpha + \beta x_t + u_{t+1}$$
$$x_{t+1} = \theta + \rho x_t + v_{t+1}$$
$$\Sigma = \begin{pmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{pmatrix}$$

where $[u, v] \sim \mathrm{N}(0, \Sigma)$ follows a multivariate normal distribution, and the following are the ***true*** values of the parameters:

Parameter	Value
α	0.01
β	0.05
θ	0.01
ρ	0.3
σ_u^2	0.6
σ_v^2	0.5
σ_{uv}	-0.5

Question 1 (4 points)

Create a function (routine) that given a set of parameters, and a sample size T simulates the dynamics of the system above. The function should return both time series y_t and x_t. Assume that both processes begin at their unconditional mean, e.g.

$$x_0 = \frac{\theta}{1-\rho}$$
$$y_0 = \alpha + \beta x_0.$$

Plot the dynamics of y_t and x_t for $T = 100$. **Do not forget that the residuals u_t and v_t are correlated**. Investigate how to generate correlated normal random variables in the programming language of your choice.

Question 2 (4 points)

Create a function that given simulated data y_t and x_t estimates the parameters of the system above using OLS. The function should return the estimated parameters $\hat{\alpha}$, $\hat{\beta}$, $\hat{\theta}$, $\hat{\rho}$, $\hat{\sigma}_u^2$, $\hat{\sigma}_v^2$, and $\hat{\sigma}_{uv}$. Estimate the parameters for a random path of y_t and x_t for $T = 100$. Compare the estimated parameters with the true parameters.

Question 3 (4 points)

Fix a sample size $T = 100$ and perform $N = 10000$ simulations of the system above. For each simulation estimate β. Plot the distribution (histogram) of $\hat{\beta}$. Can you see graphically that the OLS estimator is biased? (Locate in the distribution the true value of β).

Question 4 (4 points)

Now you are going to fix $N = 100$ and compute the bias of $\hat{\beta}$ for different sample sizes T. The bias is defined as

$$\text{Bias}(\hat{\beta}) = \text{E}[\hat{\beta}] - \beta$$

where $\text{E}[\hat{\beta}]$ is the expected value of $\hat{\beta}$ across the N simulations for a fixed T. Plot the bias of $\hat{\beta}$ for a reasonable amount of points (e.g. 500) in the interval [50, 1000].

Question 5 (3 points)

Fit the following regression using OLS

$$\text{Bias}_i = \gamma_0 + \gamma_1 \frac{1}{T_i} + \gamma_2 \frac{1}{T_i^2} + \epsilon_i$$

where Bias_i is the bias of $\hat{\beta}$ for a given sample size T_i. Plot the fitted regression line together with the bias. Compute the t-statistics of all the coefficients. What are the terms that are statistically significant?

Question 6 (1 point)

Plot the $O(1/T)$ terms of the bias presented in Stambaugh (1999), e.g. plot

$$\text{Bias}(\hat{\beta}) = -\frac{\sigma_{uv}}{\sigma_v^2} \frac{1 + 3\rho}{T}$$

how does it compare with the curve you obtained in question 5? E.g. compare

$$\gamma_1 \text{ vs. } -\frac{\sigma_{uv}}{\sigma_v^2}(1 + 3\rho)$$