Lesson 1A: Campbell and Shiller Decomposition

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Empirical Asset Pricing

The Equity Premium

- Average returns on stocks r_m are higher than the returns on short-term nominal bonds r_f.
- The Equity premium $\mathbb{E}[r_m - r_f]$ and Sharpe ratio for the U.S. is robust across samples. For a long sample from 1926.7-2021.7, the equity risk premium in the U.S. is 8.3%, return volatility is 18.5% and the Sharpe Ratio is 0.45.
- The Equity risk premium is similarly large for Europe and Asia Pacific, excluding Japan.
- Japan is a surprising **outlier** with no equity risk premium, bonds and stocks have had almost the same expected return.

The Equity Premium (Cont.)

- Equity returns are volatile, which makes it challengin to estimate the equity premium precisely. Even with 95 years of data we have a standard error of $18.5/\sqrt{95} \approx 2\%$. A 95% confidence interval ranges from 4% to 12% (t-statistics of around 2 $\times$ the standard error.)
- Avdis and Wachter (2017) provide unconditional maximum likelihood estimators of the equity risk premium.

Time-series predictability and excess volatility

- Campbell and Shiller (1988) develop a log-linear approximation of returns that results in a useful accounting identity to udnerstand the link between stock prices, fundamentals and expected returns.

$$r_{t+1} = \log\left(\frac{P_{t+1} + D_{t+1}}{P_t}\right) = \Delta d_{t+1} - pd_t + \log\left(1 + \frac{P_{t+1}}{D_{t+1}}\right)$$

where $pd_t = \log\left(\frac{P_t}{D_t}\right)$ is the log price-dividend ratio, and $\Delta d_{t+1} = \log\left(\frac{D_{t+1}}{D_t}\right)$ is the log dividend growth rate.

Campbell-Shiller decomposition

Apply a first order Taylor expansion to the last term

$$\log\left(1+\frac{P_{t+1}}{D_{t+1}}\right) \approx \kappa_0 + \kappa_1 pd_{t+1}$$

where

$$\kappa_1 = \frac{e^{\bar{pd}}}{1+e^{\bar{pd}}}$$

and

$$\kappa_0 = \log(1+e^{barpd}) - \kappa_1 \bar{pd}$$

We can approximate returns as

$$r_{t+1} \approx \kappa_0 + \Delta d_{t+1} + \kappa_1 pd_{t+1} - pd_t$$

Campbell-Shiller decomposition (Cont.)

Iterate forward

$$pd_t = \frac{\kappa_0}{1-\kappa_1} + \sum_{j=0}^{\infty} \kappa_1^j \Delta d_{t+j} - \sum_{j=0}^{\infty} \kappa_1^j r_{t+j}$$

And the transversality condition

$$\lim_{j \to \infty} \kappa_1^j \mathbb{E}[pd_{t+j}] = 0$$

Note: You can use that transversality condition to test for bubbles. Giglio, Maggiori and Stroebel (2016) use this approach to test for bubbles over 700 years of data in the UK

Present Value Relation

The equation holds ex-post as well as ex ante.

$$pd_t = \frac{\kappa_0}{1-\kappa_1} + \underbrace{\mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j \Delta d_{t+j}}_{\Delta d_t^H} - \underbrace{\mathbb{E}_t \sum_{j=0}^{\infty} \kappa_1^j r_{t+j}}_{r_t^H}$$

- Movements in prices can be attributed to fluctuations in exected growth rates, expected returns or both.

Present-value relation (variances)

$$Var(pd_t) = Var(\Delta d_t^H) + Var(r_t^H) - 2Cov(\Delta d_t^H, r_t^H)$$

Expected discounted future dividend growth rates or returns have to be volatile or they have to be negatively correlated if prices are to be volatile.

- Shiller (1981) provides the first evidence that prices appear to move more than what is implied by expected dividends, even realized dividends. This is the celebrated excess volatility puzzle.

Present-value relation (covariances)

$$\begin{aligned} Var(pd_t) &= Cov(\Delta d_t^H - r_t^H, pd_t) \\ &= Cov(\Delta d_t^H, pd_t) - Cov(r_t^H, pd_t) \\ 1 &= Cov(\Delta d_t^H - r_t^H, pd_t) \\ &= \frac{Cov(\Delta d_t^H, pd_t)}{Var(pd_t)} - \frac{Cov(r_t^H, pd_t)}{Var(pd_t)} \end{aligned}$$

- First term is the slope of a regression predicting future dividend growth rates with the price-dividend ratio.
- Second term is the slope of a regression predicting future returns with the price-dividend ratio.
- he sum of both slopes has to be equal to one. The dog that did not bark (Lettau and Van Nieuwerburgh, 2008 and Cochrane, 2008)

Empirical Evidence

- Typical empirical framework ($pd_t = -dp_t$)

$$\begin{aligned} \Delta d_{t+1} &= a_d + \kappa_d dp_t + \epsilon_{d,t+1} \\ r_{t+1} &= a_r + \kappa_r dp_t + \epsilon_{r,t+1} \\ dp_{t+1} &= a_p \phi dp_t + \epsilon_{p,t+1} \end{aligned}$$

Coefficient Restrictions

We know that approximately

$$r_{t+1} = \kappa_0 + \Delta d_{t+1} - \kappa_1 dp_{t+1} + dp_t$$
$$r_{t+1} = \kappa_0 + \Delta d_{t+1} - \kappa_1 (a_p \phi dp_t + \epsilon_{p,t+1}) + dp_t$$
$$r_{t+1} - \Delta d_{t+1} = (1 - \phi\kappa_1) dp_t + \kappa_0 - \kappa_1 (a_p + \epsilon_{p,t+1})$$
$$Cov(r_{t+1}, (1 - \phi\kappa_1) dp_t) - Cov(\Delta d_{t+1}, (1 - \phi\kappa_1) dp_t) = (1 - \phi\kappa_1)^2 Var(dp_t)$$

The present value relation implies a coefficient restriction of the form

$$\kappa_r - \kappa_d = (1 - \phi\kappa_1)$$

Why is it different from one?