Problem Set 2

Juan F. Imbet Ph.D.

Empirical Asset Pricing
M2 104
Paris Dauphine - PSL

The problem set together with the code needs to be emailed to `juan.imbet@dauphine.psl.eu` **before March 7, 23:59**. You can solve the problem sets in groups of maximum *2* people. If you cannot email it, use a we transfer link.

Setup

Consider the following factor model with 5 assets and 2 factors with the appropriate dimensions of the parameters:

$$\underbrace{R_t^e}_{5\times 1} = \underbrace{a}_{5\times 1} + \underbrace{\beta}_{5\times 2}\underbrace{f_t}_{2\times 1} + \underbrace{\epsilon_t}_{5\times 1}$$

with $\epsilon_t \sim \mathcal{N}(0, \Sigma)$, where Σ is a non-diagonal covariance matrix. And $f_t \sim \mathcal{N}(\mu_f, \Sigma_f)$, where Σ_f is the covariance matrix of the factor realizations, and μ_f is the expected value of the factor returns.

The true values of the parameters are:

$$a = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0.5 & 0.0 \\ 0.0 & 0.5 \\ 0.5 & 0.5 \\ 0.3 & 1.2 \\ 0.7 & 0.4 \end{pmatrix},$$

$$\Sigma = \begin{pmatrix} 1.0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1.0 & 0.5 & 0.0 & 0.0 \\ 0.5 & 0.5 & 1.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 0.0 & 1.0 & 0.5 \\ 0.5 & 0.0 & 0.0 & 0.5 & 1.0 \end{pmatrix}$$

and

$$\mu_f = \begin{pmatrix} 0.05 \\ 0.07 \end{pmatrix}, \quad \Sigma_f = \begin{pmatrix} 1.0 & 0.5 \\ 0.5 & 1.0 \end{pmatrix}$$

Question 1 (4 points)

Create a function that given a time horizon T simulates the dynamics of the system above. The function should return both time series R_t^e and f_t.

Question 2 (4 points)

Create a function that given simulated data R_t^e estimates using OLS and GLS the parameters $\hat{\alpha}$ and $\hat{\lambda}$ (together with their standard errors) in the following model (assume that you know the true values of Σ, Σ_f and β so you don't need to estimate them):

$$E_T[R_t^e] = \alpha + \beta\lambda$$

Question 3 (4 points)

For a given $T = 10$ repeat the above exercise 1000 times and plot the distribution of the estimated parameters $\hat{\alpha}$ and $\hat{\lambda}$ (together with the true values). What do you observe? The true values of α are a and the true values of λ are μ_f.

Question 4 (4 points)

Repeat the above exercise, but for each estimated parameter consider the distribution of the ratio

$$\frac{\hat{\theta} - \theta}{\text{s.e.}(\hat{\theta})}$$

Can you find any difference in the distribution of the ratios between OLS and GLS? Hint: look at the tails of the distribution. Comment on the results.

Question 5 (4 points)

Assume now that you do not know the true values of β, Σ and Σ_f. For a fixed $T = 10$ and 1000 simulations, compare the expected value of your estimators. Does estimation error affect the expected value of $\hat{\alpha}$ and $\hat{\lambda}$? Comment on the results.